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Solving the Travelling Salesman Problem Using Spreadsheets

By Stelios Nikolakakis, Doctoral Candidate

The main concept of the Travelling Salesman problem (TSP), is to find the minimum length tour, given a specific number of cities, by visiting every city only once, and then returning back to the starting city. In bibliography, the starting city can also be found as depot. Many real world applications can be modeled as a TSP, including the computer wiring problem, overhauling gas turbine engines in aircraft, and job scheduling on a single machine.

TSP variants

The TSP has drawn much of attention, due to its variations. The Asymmetric Travelling Salesman Problem (ATSP) [1] is when the cost of travelling from city i to j, is different than the cost from j to i. Moreover, in the Multisalesman problem (MTSP) [2] can be found more than one salesman, serving the clients, while in the Maximum Benefit Travelling Salesman Problem (MBTSP) [3] a benefit is associated with each visited city. Finally, the Time Dependent Travelling Salesman Problem (TDTSP) [2] includes time periods during which each customer has to be served.

Application

TSP and its variations are considered to be non deterministic polynomial-time hard problems (NP hard). This means that, solving them with the standard Linear Programming/Mixed Integer Programming (LP/MIP) solvers, can only be achieved in relatively small problems. However, the latest advances in solver engines, combined with the exploitation of the features that the spreadsheets offer, present a great opportunity for solving highly sized problems. It is the flexibility and the layout of spreadsheets that offer the opportunity for a clear problem formulation, as well as an explicit communication of the solution.

A very descriptive and comprehensive approach on modeling and solving TSP through spreadsheets, is shown in figure 1 [4]. There are ten oil rigs that are to be served by a supply ship. The coordinates of each oil rig is shown are shown in the figure, from cell B2 to D13. Additionally, the Euclidean distances between the oil rigs are shown from cell G3 to cell Q13. In the lower part of the spreadsheet can be seen the sequence of the oil rigs in the final solution, as well as the coordinates of each part of it, and a graphical representation. The final solution is generated through the Premium Solver Platform, in which a new feature is added. The only constraint that has to be specified, is that all variables in the final solution are different. That means, each time a new oil rig is added in the solution, it has to be different from those that already exist, and moreover, it has to minimize the total traversal cost. Finally, the objective in the solver settings is to minimize the 'Total Cost' cell (D28). The optimum sequence is 0-3-1-10-8-2-7-5-6-4-9-0.

The latter application presented, can not be applied in large problems, due to the non-smooth nature of the objective function, by using functions like *Index* in spreadsheets. Different formulations, more flexible, but with the same main concept, can be produced for variants of the TSP, such as in non complete graphs, with open or closed tours, or multiple visits [4]. However, an interesting approach, that could be studied in the future, would be to add time window constraints in each of the customers/oil rigs that are to be visited. This means that not only the final permutation would appear, but also the exact time window in which the node has to be served.

References

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