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## The Revelation Principle: a Game Theory touch upon Supply Chain Management Problems

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An increasing body of literature in the area of supply chain management addresses the way in which the nodes of the chain (players) can act in a cross-linked mode in order to reduce both their own costs as well as the total cost of the supply chain (in terms of inventories, transportation, ordering, etc.). Players may have complete (in the majority of the approaches proposed to-date) or incomplete (asymmetric) information; the latter assumption may lead to different and divergent strategies and tactics, and such a hypothesis is more realistic in practice. One way to infuse correlation in supply chain interactions is to use incentive schemes, an example being the typical quantity discounts of EOQ models.

In practice it is very usual, for some or all the players to have private information about the supply chain in which they participate. Consider for example a chain with two nodes (a supplier and a buyer) that trade a single product, in which the supplier produces in a lot-for-lot fashion. The buyer knows exactly his own cost structure while the supplier is aware only of a distribution function about the buyer's cost structure. This may occur because the buyer operates in a different country or in geographical isolation with respect to the supplier. We allow players to communicate with each other regarding any private information they may have, as a way to coordinate their actions. The communication is free, without any contracts, and the players may hinder the truth regarding the information they possess. Also, it is very important that all players are free to choose their strategies and tactical moves, without making any coalition, while each player chooses his own strategy in order to minimize his (expected) cost, after an initial inter-node communication. Within this framework, examining all possible means and ways for communication between the players might be intractably complex.

We assume that players in the supply chain can communicate via a mediator (third trusted party), a fact that has the following attributes: First, the mediator announces a plan which describes his role and defines his potential actions (referred to as the 'mediator plan'). Then, each player reports confidentially his private information to the mediator; the player is free to report anything (true or lie), in order to minimize his cost (from his own perspective). After receiving all the reports from all supply chain nodes, the mediator specifies actions for the players, according to the mediator plan, and confidentially recommends the actions to the players. Note that a mediator plan incorporates any rule that emanates from the players' reports and enables the specification of actions (and their associated probabilities) that the mediator could recommend to the players. Finally, the players select their own actions in order to better serve their own goals. A key requirement of this framework is the existence of a mediator who is completely trustworthy and incurs no cost to the supply chain. This hypothesis is not restrictive for a supply chain, since this role can be assumed by third party logistics providers, government regulating authorities or other organizations that possess such characteristics.

Obviously, opportunities for mutual benefits cannot be found, unless the players share their private information. To proceed in sharing private information, players must be provided with appropriate incentives. Towards this end, the Revelation Principle (RP), which is a tool of Game Theory, offers significant insights. The RP was first proposed by Gibbard (1973) and Myerson (1991), Hurwicz and Maskin, who extended the RP's attributes and were awarded the 2007 Nobel Prize in economics. The RP is a technical approach that allows the derivation of statements about what rules are feasible in a communication system. The RP asserts that, any equilibrium of a communication system can be reached by an equivalent incentive-compatible (IC) mechanism. In other words, if there exist equilibriums in a communication system, there always exist IC-mechanisms that can reach them. An IC-mechanism is a mediator plan, in which the mediator includes two kinds of incentives: the incentive of adverse selection (in order to give players an incentive to tell the truth to the mediator about their private information) and the incentive of moral hazard (in order to give players an incentive to choose the actions which they have been recommended by the mediator).

A mechanism belongs to the IC class if, under the hypothesis that the other players are honest and obedient to the mediator, no player could ever expect reduced cost by dishonestly reporting information to the mediator or by ignoring the mediator's recommendations, or by a combination of the two. The benefit from the RP is that it guarantees that it is sufficient to consider only the IC-mechanisms when devising the mediator plan. This class of mechanisms has the appropriate mathematical properties that both simplify the associated mathematical analysis (because the IC-mechanism can be defined by a combination of two finite sets of linear inequalities) and allow us to make general statements about any communication system.

In the example of one buyer and one supplier introduced above, only the buyer has private information. We consider that the buyer's holding cost (h) is either h_L with probability p or h_H with probability 1-p. Without loss of generality, we suppose that h_L<h_H. The supplier has to decide what quantity discounts (i.e., a function of quantity P(Q)) should propose to the buyer. The supplier is not aware of the actual buyer's holding cost, but he knows the probabilities p and 1-p. In contrast, the buyer has to decide what quantity (Q) he should order. Thus, the cost structure for the two nodes is:

$$C_B(Q,P) = K_B \cdot D/Q + 1/2 \cdot h \cdot Q - P(Q), \text{ and } C_S(Q,P) = K_S \cdot D/Q + P(Q),$$

where: C_B ,C_S are the total costs of the buyer and the supplier respectively, K_B,K_S are the setup costs and D is the demand for the time period considered in the system. Each player makes his choice (the buyer chooses Q, while the supplier chooses the function P(Q)) in order to minimize his (expected) cost. It is known that if the players cooperate, they achieve a joint optimal point, where both reduce their overall costs. The assumptions both of asymmetric information and of the fact that the nodes do not make coalition are crucial. We allow the players to communicate via a mediator, with respect to actual buyer's holding cost, in a way to correlate and achieve a better payoff. The key point is that, according to the RP, if the buyer reports honestly his holding cost (private information) to the mediator, and both the buyer and the supplier follow the mediator's recommendations, an equilibrium is reached and the overall cost of the supply chain with respect to inventory holding is reduced versus what can be achieved if the mediator was not present.

### References

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