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Stock Index Realized Volatility Forecasting in the Presence of Heterogeneous Leverage Effects and Long Range Dependence in the Volatility of Realized Volatility

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"A volatility model must be able to forecast volatility; this is the central requirement in almost all financial applications" (Engle and Patton, 2001, p.1). Indeed, everyday core business functions such as Basel II capital adequacy calculations, risk management, capital allocation, derivatives pricing and hedging, rely on accurate volatility estimation and forecasting. A plethora of volatility implementations have been proposed in the open literature see, e.g., Poon and Granger (2003) for a good review. In Andersen and Bollerslev (1998), the authors showed that the daily unobserved volatility could be adequately approximated by the sum of squared intraday returns, the so-called, realized volatility[1]. As evidence appeared that realized volatility possessed long memory, a number of researchers employed the Autoregressive Fractionally Integrated Moving Average (ARFIMA) specification for its modelling (e.g. see Andersen *et al*., 2003; Giot and Laurent, 2004; Koopman *et al*., 2005; Degiannakis, 2008; Angelidis and Degiannakis 2008; Martens *et al*., 2009).

An alternative implementation, based on the Heterogeneous Market Hypothesis and Muller's *et al*. (1997) which states that market agents differ with respect to their investment horizon, risk aversion, degree of available information, institutional constraints, transaction costs, etc. This diversity is identified as the root cause of asset volatility, as market agents aim to settle at different asset valuations, according to their individual market view, preferences and expectations. A first attempt to model the volatility dynamics of a heterogeneous financial market was made by Muller et al. (1997) who proposed the Heterogeneous ARCH (HARCH) model. The HARCH is an ARCH type model which accounts for the distinct volatility behavior at different time resolutions using lagged squared returns of different time horizons (for practical applications of the HARCH model see McMillan and Speight, 2006a; McMillan and Speight, 2006b).

Based on the Heterogeneous Market Hypothesis and the HARCH model, Corsi (2009) proposed an approximate long memory model for realized volatility, the Heterogeneous Autoregressive Realized Volatility model, denoted as HAR henceforth. The author suggested that a significant contributor to the market's heterogeneity was the presence of three types of market agents with different time investment horizons: short (daily), medium (weekly) and long term (monthly) investment horizons. Short-term traders (such as hedge funds, FX and statistical arbitrage traders) typically adjust their market positions intradaily, swiftly reacting on any relevant new information. Medium and long-term investors (such as commercial banks and pension funds) have longer holding periods and restructure their trading portfolios according to lower frequency information flow. Hence, the same informational content is distinctly assimilated across market participants, inducing different reaction times to the same market events. This asymmetry leads to a hierarchic structure of volatility components with distinguishable frequencies, where low (e.g. monthly) frequency volatility components should yield a greater impact on the overall volatility than high (e.g. daily) frequency volatility components. The economic rationale is that short-term investors interpret the level of long-term volatilities as predictions of future volatility and adjust their trading strategies accordingly, while short-term volatility is irrelevant to investors with longer holding periods

We contribute to this growing literature by introducing a logarithmic HAR model with asymmetries, or leverage effects[2], modelled as lagged standardized returns and absolute standardized returns (analogous to an EGARCH-type structure), occurring at distinct time horizons: daily, weekly and monthly. Moreover, in order to capture any remaining long range dependence in the volatility of realized volatility, we propose a Fractionally Integrated GARCH (FIGARCH) implementation for the conditional heteroscedasticity of the residuals. We also apply the Realized Power Variation (RPV), proposed by Barndorff-Nielsen and Shephard (2004) as a regressor, which has been shown to be a robust to jumps, more persistent and accurate predictor of future volatility than realized volatility.

Estimation results (using two ten year data sets from the S&P 500 and DJIA stock indices) reveal that not only past negative daily, but also weekly and monthly negative shocks yield a greater impact on current volatility than positive ones, suggesting a heterogeneous component structure in asymmetric effects. Moreover, an interesting contribution of past monthly positive shocks is also identified. Although the inclusion of leverage effects in the HAR regression reduces both the skewness and the heteroskedasticity of the error term, it does not eliminate the ARCH effects. Through Exact Local Whittle (ELW) and Maximum Likelihood Estimation (MLE) integration order estimations, the suspected long range dependence in the volatility of realized volatility is also verified. The inclusion of the abovementioned characteristics in the volatility equation has significant practical implications, as the out-of-sample forecasting performance of the proposed specification is considerably improved compared to other HAR and ARFIMA applications. Finally, our findings are found to be robust against microstructure noise.

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[1] The sum of squared intraday returns is actually the realized variance. Realized volatility is defined as the square root of realized variance. However, many authors use the term realized volatility interchangeably with the term realized variance.

[2] "Bad news" in a stock market (i.e. negative returns) tend to increase future volatility more than "good news" (i.e. positive returns). This asymmetry between negative and positive returns is referred to as asymmetric or leverage effect. In theory, the leverage of the company increases as its stock price goes down, i.e. the company uses more debt than owned capital to finance its business activities. This increases the risk of investing in this stock which in turn increases its volatility.