May 2009

Non-parametric derivative pricing models: Weaknesses and Cures with Emphasis on Financial Risk Management
By Efstathia Korkou, Doctoral Candidate

The accurate measurement of financial risk is an issue of paramount importance for financial institutions and practitioners alike. The inadequacies and shortcomings of existing risk management models were identified as one of the root causes for the recent global financial crisis.

In practice, financial risk management is effectively carried out through complex financial instruments such as forwards, futures, swaps and options. The exact financial derivatives pricing and in particular of options, is crucial for risk management applications. Until now the parametric systems have been the norm in the pricing of derivatives. However, parametric systems are constrained by specific functional forms such as cumulative normal distributions, while they prove to be insufficient in the absence of historical data.

Black and Scholes (1973) first proposed a theoretical framework for options pricing which relied however on some highly restrictive assumptions, which were often blamed for some common pricing inconsistencies. Theses inconsistencies become more pronounced in moneyness issues and more specifically in the pricing of in-the-money and out-of-the-money options.

In option pricing, probably the most important parameter to be defined is the volatility of the underlying asset. The Black-Scholes model assumes a constant volatility behavior, an assumption which is often violated in practice. Moreover, observed biases such as the "volatility smile" or the "volatility smirk" prompted the development of more complex parametric and non-parametric methods for pricing options. Non-parametric models, in contrast with parametric ones, are not constrained by specific functional forms such as cumulative normal distributions. Hence, they are inherently more effective in "loosening up" the Black-Scholes model assumptions and restrictions.

A popular class of non-parametric pricing models are the feed forward Neural Networks (NN) which are widely acknowledged for their universal approximation abilities. Their successful implementation in financial applications and problems has been broadly recognized in the open literature, e.g.: Hutchinson et al. (1994), Qi and Maddala (1996), Liu (1996), Gençay and Salih (2003).

However, despite that non-parametric models do yield significant advances in option pricing, they suffer from inherent weaknesses such as:

- Overfitting, resulting from the universal approximation properties of the estimators. This issue can however be addressed by restricting the universal approximation properties of the estimators. This has been achieved via cross validation, the use of hints - homogeneity hint - (Garcia, Gençay, 2000), or normalization methods such as Bayesian Regulization, Early Stopping, or Bagging (Gençay, Qi, 2001).

- Robustness issues, or otherwise the inability of the learning procedure to deal with specific time series characteristics such as:
- Robustness against discontinuous non stationary datasets, where the Ordinary Least Squares (OLS) method proves inadequate.
- Robustness against noise and outliers which result in complex non-linear transformations.
- Robustness against collinearity and serial correlations, where the problems stem from colinearities between the explanatory variables and serial correlations in the residuals respectively.

- Lack in diagnostics. Although that well developed diagnostic tools such as the t-statistic, R2, or the DW statistic exist for parametric models, there is a lack of reliable diagnostic tools for non-parametric ones.

- Regarding the NN robustness issues, the researchers' attention is currently focused on the OLS minimization method. Although this approach is perfectly acceptable for environments without noise, the OLS learning procedures cannot deal with some, often quite common, time series characteristics such as those mentioned above.

Finally, the development of accurate and reliable diagnostics for non-linear models and in particular for NN applications, is still at an infancy stage mainly due to computational complexity and overhead restrictions.The recent experience attests that extremes in volatility put into question the established risk management framework. It rests also upon the academia to propose new mathematical techniques and methodologies that capture the market reality better than the long established models do today. In the research framework outlined above, the challenge is to develop a "robust" least squares methodology as well as a set of reliable diagnostic tools for the identification of well-defined non-parametric models.

References

Garcia, R. and Gençay, R.(2000), "Pricing and Hedging Derivative Securities with Neural Networks and a Homogeneity Hint", *Journal of Econometrics* 94 (1-2): 93-105.
Gençay, R. and A. Altay-Salih (2003), "Degree of Mispricing with the Black-Scholes Model and Nonparametric Cures", *Annuals of Economics and Finance* 4: 73-101.
Gençay, R. and R. Gibson (2007), "Pricing Model Risk for European-Style Stock Index Options", *IEEE Transactions on Neural Networks* 18(1): 193-202.
Gençay, R. and M. Qi (2001), "Pricing and Hedging Derivative Securities with Neural Networks: Bayesian Regulization, Early Stopping and Bagging", *IEEE Transactions on Neural Networks* 12(4): 726-734.
Aït- Sahalia, Y. and A.W. Lo (1998), "Nonparametric Estimation of State-Price Densities Impicit in Financial Asset Prices", *Journal of Finance* 53 :499-547.
Carapeto M, Holt W, Refenes APN (2003), "On model complexity and selection", *Journal of Statistical Computation and Simulation*, Vol.73, pp: 45-57.
Refenes APN and Holt WT (2001), "Forecasting volatility with neural regression: A contribution to Model Adequacy", *IEEE Transactions on Neural Networks*, Vol.12, No. 4, pp: 850-864.
Refenes APN and Zapranis AD (1999), "Neural model identification, variable selection and model adequacy", *Journal of Forecasting*, Vol.18, Issue:5.