Optimal Portfolio Allocation with Higher Moments

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The statistical properties of asset returns (equities, bonds, foreign exchange currencies) are subject to continual interest and research, mainly because the knowledge of them has important economic implications for the money market equilibrium, and the international trade including the assessment of open positions in reserves, payments of underlying assets in imports and exports area, and money lending and borrowing. Also, the wide fluctuations in stock and foreign currencies market exchanges are of continual concern to investors and to their management decisions because these changes in volatility may have adverse effect on risk management, hedging strategies and their investment decisions.

Especially it is crucial to identify properly the type of heteroscedasticity in the data-generation process and the predictive distributions, which depend on the conditional distribution for the normalized error. Accurate specification plays an important role in international finance, particularly in the area of the pricing of foreign currency options and futures and the selection of mean-variance efficient portfolios of international assets.

An extensive literature documents the behaviour of the asset returns distribution. Traditional ARMA/ARIMA models have concentrated on modeling and forecasting the conditional mean (a location parameter) assuming variance and all higher order moments to be constant. A natural extension has been to model the conditional variance (a scale parameter), so as to capture the most known characteristic of asset returns is the volatility clustering first noted by Mandelbrot (1963): Large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes. Fama (1970) also observed the alternation between periods of high and low volatility: Large price changes are followed by large price changes, but of unpredictable sign. This extension was motivated by the empirical fact that volatility in many economic time series and particularly in financial time series did not remain constant over time. Engle (1982) proposed to model time varying conditional volatility with the Autoregressive Conditional Heteroscedasticity (ARCH) processes. Bollerslev (1986) showed that high ARCH order has to be selected in order to catch the dynamic of conditional variance introducing the Generalised ARCH (GARCH) models. Since then, a wide range of GARCH formulations appeared in the literature, e.g., IGARCH, Nelson (1990), EGARCH, Nelson (1991), APARCH of Ding, Granger and Engle (1993). Several simplified versions of these models have been applied to different asset data series (e.g. Milhoj 1987, Bollerslev 1987, Hsieh 1988).

All the above models are univariate and try to estimate the properties of financial assets without considering the importance of conditional variations in moments other than the mean and variance. Moreover, an issue that has not received yet much attention in the hedging literature concerns the importance of conditional variations in moments other than the mean and variance. Higher moments, such as skewness and kurtosis, can control the asymmetries and thick tails that are typically observed in the distributions of financial returns. Their role has been widely studied within the context of risk management and portfolio theory. A number of authors, including Lai (1991), Prakash et al. (2003), Sun and Yan (2003), and, Jondeau and Rockinger (2006), have provided evidence suggesting that the incorporation of higher moments in portfolio allocation leads to superior approximations of expected utility and allows a very efficient way to compute optimal portfolios. Even if the first two moments are sufficient for asset pricing, higher moments are potentially important for hedging since variations in the shape of the distribution may affect the estimation of the conditional mean and variance. Although models from the GARCH family are able under certain assumptions and parameterisations to produce thick-tailed and skewed distributions, they typically assume that the shape parameters are time invariant. Different models have been developed in the literature in order to capture dependencies in higher moments (eg., see Gallant, Hsieh and Tauchen, 1991; Hansen 1994; Harvey and Siddique, 1999, 2002a, 2002b; Jondeau and Rockinger (2006); Brooks, Burke and Persand, 2005), but their usefulness for optimal hedge ration estimation has not been yet investigated.

The goal of the related research in the field of optimal hedging and higher moments is to extend the literature on portfolio selection with higher moments by investigating how non-normality of returns and higher moments may affect hedging strategies. In order to account for time varying skewness and kurtosis in optimal hedge ratio estimation the ARCD model proposed by Hansen (1994) is employed. In a horserace of models, the dynamic hedging effectiveness of the ARCD is compared to that obtained by OLS, error-correction, exponential moving averages, and, univariate and multivariate GARCH. Effectiveness is measured in-sample and out-of-sample using the minimum variance method. Spot and futures daily closing prices are used for stock indices from the US, UK and Germany. The results suggest that the hedging performance using the ARCD outperforms that obtained by the competing approaches.

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