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Realized Volatility and Risk Management

By Dimitrios Louzis, Doctoral Candidate

Volatility in financial markets has been extensively investigated due to its paramount importance in financial economics and risk management. Since Engle (1982) proposed the Autoregressive Conditional Heteroscedasticity (ARCH) model, a plethora of prominent econometricians have been working on modeling time varying variances and covariances. Recently, Andersen et al (2000, 2001a, 2001b, 2003, 2005) proposed using realized volatility as an accurate estimation of daily integrated volatility. Realized volatility grants new insights to volatility modeling as it does not presume an explicit model for daily volatility.

Typically, realized daily volatility is defined as the sum of squares and cross products of high frequency intra-day returns (Andersen et al, 2001a). From the above definition it is obvious that the risk analyst is not required to estimate a parametric GARCH type model (originally developed by Engle (1982) and Bollerslev (1986)) with specific distributional assumptions in order to obtain estimates of the daily variances and covariances. Daily realized volatility is model free and utilizes all the information embedded in intra-day asset price movements (Andersen et al, 2000, 2001a, 2001b, 2003, 2005). The availability of high frequency intra-day data, allows the idea of realized volatility to be readily applied in practical risk management i.e. to the calculation of daily Value at Risk (VaR). However, the selection of the time intervals used in the calculation of daily realized variance is of crucial importance.

In theory, as the sampling frequency increases, the computed realized volatility converges to the true daily integrated volatility of the underlying continuous time process (Andersen et al, 2005). In practice however, small sampling time intervals are plagued with market microstructure issues such as price discreteness, infrequent trading and bid - ask bounce effects. Minimization of the aforementioned biases is achieved by choosing an appropriate sampling frequency of high quality intra-day prices (from 5 to 30 minute intervals, depending on the asset). Thus, the risk manager can treat realized volatility as an observation of the real integrated daily variance; using standard time series techniques (i.e. ARMA or ARFIMA models,) it is viable to forecast the day ahead volatility as in Andersen, et al (2000, 2001a, 2001b, 2003). Their results suggest that realized variance methods yield superior forecasts compared with GARCH style models.

The multivariate generalization of realized volatility is the realized Variance Covariance (VCV) matrix. Its parametric counterparts, i.e., the multivariate GARCH models (for a detailed survey on multivariate CARCH models see Bauens et all, 2006), suffer from the problem of dimensionality. Multivariate GARCH models are computational burdensome and they do not ensure that the VCV matrix is positive definite, thus hindering the implementation of multivariate GARCH models in practice. Again, treating realized variances and covariances as observations, one can use standard multivariate time series techniques, e.g., Vector Autoregressive (VAr) models in order to produce accurate forecasts of the VCV matrix. VAr models are more flexible and easy to estimate than their variance counterparts, the multivariate GARCH models. Moreover, the positive definiteness of the VCV matrix is guaranteed as long as the asset returns are linearly independent and the number of assets does not increase relatively to the sampling frequency of the intraday returns (Andersen et. al, 2003). Since the above conditions can be easily violated, especially in high dimensionality systems, Andersen et al (2005) proposed the reduction of the dimensionality of the VCV matrix by mapping to liquid base assets which are considered to be the key drivers of risk.

It is hence interesting to investigate how realized volatilities and correlations can be applied into practical risk management applications. Although previous research focused on using density forecasts in order to obtain VaR estimates (Andersen et al, 2003, Giot and Laurent, 2004), the same forecasts can be used for calculating the probability of loss exceeding a specified threshold, i.e., shortfall probabilities, or the expected loss conditional upon loss exceeding a pre-specified threshold, i.e., the expected shortfall. Limited previous work suggests that realized volatility techniques produce superior (Andersen et al, 2003, Giot and Laurent, 2004) VaR estimates compared with GARCH family models and industry models (i.e. RiskMetrics' Integrated GARCH). The next step in VaR analysis is to consolidate the nonlinearities of realized volatility. There is evidence that realized volatility non-linear models could produce even better volatility forecasts. Finally, the effectiveness, functionality and forecasting ability of realized volatility techniques in high dimensional settings has not yet been studied in detail. One critical issue is to ensure the positive definiteness of the VCV matrix. One possibility could be the direct modeling of Cholesky factors (a mathematical reparameterization of the VCV matrix) rather than the realized VCV matrix itself. Accurate and computationally efficient estimation of the structure of the VCV matrix is of crucial importance in many practical financial applications such as asset allocation, portfolio management and asset pricing.

References

Andersen, T. G., Bollerslev, T., Diebold, F. X and Labys, P. (2000). Exchange rates standardized by realized volatility are (nearly) Gaussian, Multinational Finance Journal, 4, 159-79.

Andersen, T. G., Bollerslev, T., Diebold, F. X and Ebens, H. (2001a). The distribution of realized stock return volatility, Journal of Financial Economics, 61, 43-76.

Andersen, T., Bollerslev, T., Diebold, F., Labys, P., (2001b). The distribution of realized exchange rate volatility. Journal of the American Statistical Association 96, 42- 55.

Andersen, T. G., Bollerslev, T., Diebold, F. X., & Labys, P. (2003). Modeling and forecasting realized volatility. Econometrica, 71, 529-626.

Andersen, T. G., Bollerslev, T., Christoffersen P. F., Diebold, F. X (2005a). Practical volatility and correlation modelling for financial market risk management, Risks of Financial Institutions.

Bollerslev, T., (1986). Generalized autoregressive conditional heteroskedasticity. Journal of Econometrics 31, 307-327.

Bauwens, L., Laurent, S., Jeroen. V., Rombouts. K., (2006). Multivariate GARCH models: a survey, Journal of Applied Econometrics 21, 79 - 109.

Giot. P., Laurent. S., (2004). Modelling daily Value - at - Risk using realized volatility and ARCH type models. Journal of Empirical finance, 11, 379 - 398.