# The Role of Higher Moments in Time Varying Conditional Covariances

**By Eleftheria Kostika, Doctoral Candidate**

Understanding and estimating time varying conditional variances and covariances is important for many issues in finance since there are many applications that rely on multivariate covariance models. It is essential, for optimal hedging, asset allocation, derivatives pricing and risk management, the accurate modelling and forecast of the assets returns co-movement. Bollerslev, Engle and Wooldridge (1988), Cecchetti (1988), Myers and Thompson (1989), Baillie and Myers (1991), Kroner and Sultan (1993), Ng and Kroner (1998), argue that financial prices are characterized by time varying variances and covariances, presenting a variety of multivariate GARCH models. Bollerslev, Engle and Wooldridge (1988) suggested the VEC model and the diagonal VEC (DVEC) model in which the variances depend only on their past squared errors and the covariances on their past cross-products of errors.

Given the excessively large parameters needed to estimate the VEC model and the necessity to impose strong restrictions on the parameters Engle and Kroner (1995) proposed the BEKK parametrization avoiding unrealistic assumptions such as that the correlation between the conditional variances is constant (Constant correlation model by Baillie and Bollerslev 1990), and guaranteeing that the time varying covariance matrix is positive definite. Additional models can be found in Engle, Ng and Rothschild (1990b) who proposed factor models (FGARCH), in Alexander and Chibumba (1997) who proposed the orthogonal GARCH models (O-GARCH), in Tse and Tsui (2002) and of Engle (2002) who suggested the Dynamic Conditional Correlation models (DCC). All the above models assume that asset returns are jointly normally distributed ignoring the fact of asymmetry in volatility and covariance, fat tailness and skewness. However, asymmetry and skewness in distribution, is found in many financial assets since their return distributions depart far away from normality. For instance, French, Schwert and Stambaugh (1987) rejected normality claiming significant conditional skewness in daily residuals of the SP500 returns, Hong (1988) found abnormally high kurtosis in daily NYSE stock returns, Harvey (1995) observed deviations form normality in emerging stock markets indices, Harvey and Siddique (1999) showed that conditional skewness is important and consistent with asymmetric variance in daily, weekly and monthly returns of selected markets.

Since there is well established stylized evidence that financial returns exhibit fat tails and skewness, a lot of studies focused on using of non normal distributions to better model this excess kurtosis and skewness. More specifically, in the univariate framework, a large variety of conditional densities has been employed to accommodate the asymmetry and fat tailness. Hansen (1994) was the first to propose a Skew-Student distribution which allows for conditional higher moments. Recently, Harvey and Siddique (2002a, 2002b), Jondeau and Rockinger (2006) and Yan (2005), Brooks, Burke and Persand (2005) among others, have discussed ways to jointly estimate time varying conditional variance and skewness, but their resulting formulation is difficult to be implemented, moreover in a multivariate extension.

More precisely, none of the popular multivariate models are compatible with the skewness and kurtosis of asset returns since they assume multivariate normality. A few studies exist on the higher moments modelling in multivariate approaches. Harvey, Ruiz and Shepard (1994) and Fiorentini, Sentana and Galzolari (2000) replace multivariate Gaussian density with student density by letting conditional innovations to follow a Student-t distribution. Sahu, Dey, and Branco (2003), and Bawens and Laurent (2005) propose a multivariate skew Student density with support on the full Euclidian space. Their main finding is that this density improves the quality of out of sample VaR forecasts. More recently, Hafner and Rombouts (2004) and Rombouts and Verbeek (2005) apply a multivariate semi parametric GARCH estimation technique to capture higher moments showing that in within sample portfolios' VaR the model's superiority and robustness is confirmed. Azzallini (1996) and De Luca, Genton and Loperfido (2006) propose the multivariate Skew-GARCH model including a parameter to control skewness. Lee and Long (2006) introduce copula-based multivariate GARCH, the C-MGARCH with uncorrelated dependent errors, arguing that in terms of in sample model selection and out of sample multivariate density forecast, the choice of copula functions is more important than the volatility models. The main drawback of the above models is that are rather complex, and suffer from a large parameters estimation and convergence problems.

The goal of the related research in the field of higher moments and its implications in the conditional variances and covariances processes, conducted within the Management Science Laboratory, is to propose an alternative, simplified multivariate model, the simplified Multivariate Autoregressive Conditional Density Model (S-ARCD) which is compatible with the skewness and kurtosis of the financial returns and is easy to be implemented increasing the computational efficiency. It is based on the Autoregressive Conditional Density Model (ARCD) proposed by Hansen (1994) and involves the estimation only of the univariate specification of the above model. The conditional variances are calculated by the simple univariate models, and the conditional covariance is then imputed from these variance estimates. In order to evaluate the performance of the new S-ARCD model, the VaR of different portfolios is forecasted and it is compared to the VaR forecasts estimated from the ad hoc multivariate version of GARCH (Wang, Yao, 2005) and BEKK models. The results, using both statistical and economic criteria, suggest that the simplified multivariate version of ARCD performs at least well as the other two models indicating the higher moments' importance in volatility forecasting and VaR calculation.

## REFERENCES


- Alexander, C., & Chibumba A., (1997). Multivariate Orthogonal Factor GARCH, University of Sussex, Mimeo.
- Azzalini, A. & Dalla Valle, A. (1996). The multivariate skew normal distribution, Biometrika, 83, 715-726.
- Baillie, R.T. and Bollerslev, T. (1990). The Message in Daily Exchange Rates: A conditional Variance tale, Journal of Business and Economic Statistics, 7, No. 3, 60 - 68.
- Baillie, R.T., & Myers, R. (1991). Bivariate GARCH estimation of the optimal commodity futures hedge, Journal of Applied Econometrics, 6, 109-124.
- Bauwens, L., & Laurent, S. (2005). A new class of multivariate skew densities, with application to generalized autoregressive conditional heteroscedasticity models. J. Bus. Economic Statist. 23, 346-354.
- Bollerslev T., Engle R., & Wooldridge J, (1988). A Capital Asset Pricing Model with Time-Varying Covariances, Journal of Political Economy, 96 (1), 116-131.
- Brooks, C., Burke, S.P., & Persand, G. (2005). Autoregressive Conditional Kurtosis. Journal of Financial Econometrics 3, 339-421.
- Cecchetti, S., Cumby, R., & Figlewski S. (1988). Estimation of the optimal futures hedge, The Review of Economics and Statistics, 70, 623-630.
- De Luca G., Genton M. & Loperfido N. (2006). The Multivariate Skew GARCH model, Advances in Econometrics: Econometric Analysis of Financial and Economic Time Series, 20, 33-56.
- Engle, R.,(2002). Dynamic conditional correlation - A simple class of multivariate GARCH models, Journal of Business and Economic Statistics, 17, 239-250.
- Engle, R., & Kroner K., (1995). Multivariate simultaneous generalised ARCH, Econometric Theory, 11, 122-150.
- Engle R., Ng V., & Rothschild M., (1990b). Asset pricing with a factor-ARCH covariance structure, Journal of Econometrics, 45 235-7.
- Fiorentini, G., Sentana, E. & Calzolari, G., (2000). The Score of Conditionally Heteroskedastic Dynamic Regression Models with Student T Innovations, and an LM Test for Multivariate Normality, Papers 0007, Centro de Estudios Monetarios Y Financieros.
- French K., Schwert W., & Stambaugh R., (1987). Expected stock returns and volatility, Journal of Financial Economics 19, 3-30.
- Hafner, C., & J. Rombouts (2004). Semiparametric Multivariate Volatility Models," Econometric Institute Report 21, Erasmus University Rotterdam.
- Hansen, B.E. (1994). Autoregressive conditional density estimation. International Economic Review, 3, 705-730.
- Harvey, Campbell R., (1995). Predictable Risk and Returns in Emerging Markets, Review of Financial Studies, 8, 773-816.
- Harvey, C., Ruiz, E., & Shephard, N., (1994). Multivariate stochastic variance models, Review of Economic Studies, 61, 247-64.
- Harvey, C., & Siddique, A. (1999). Autoregressive conditional skewness. Journal of Financial and Quantitative Analysis, 34, 465-487.
- Harvey, C., & Siddique, A. (2002a). Time-varying conditional skewness and the market risk premium. Research in Banking and Finance, 1, 25-58.
- Harvey, C., & Siddique, A. (2002b). Conditional skewness in asset pricing tests. Journal of Finance, 55, 1263-1295.
- Hong, C., (1988). Options, Volatilities and the Hedge Strategy, Unpublished Ph.D. diss., University of San Diego, Dept. of Economics.
- Jondeau, E., & Rockinger, M. (2006). Optimal portfolio allocation under higher moments. Journal of European Financial Management, 12, 29-55.
- Kroner, K. and Ng V., (1998). Modeling asymmetric comovements of asset returns, Review of Financial Studies, 11, 817-44.
- Kroner F., & Sultan J. (1993). Time varying distributions and dynamic hedging with foreign currency futures, Journal of Financial and Quantitative Analysis, 28, 535-551.
- Lee, T.H., & X. Long, (2005). Copula-based Multivariate GARCH Model with Uncorrelated Dependent Errors, University of California, Riverside, working paper series.
- Myers, R., & Thompson, S., (1989). Generalized Optimal Hedge Ratio Estimation, American Journal of Agricultural Economics 71, 858-868.
- Verbeek, M., & Rombouts,J., (2005). Evaluating Portfolio Value-at-Risk using Semi-Parametric GARCH Models, Computing in Economics and Finance, 40, Society for Computational Economics.
- Sahu, S. K., Dey, D. K. & Branco, M. D. (2003). A new class of multivariate skew distributions with applications to Bayesian regression models. Canadian Journal of Statistics, 31, 129-150.
- Tse, Y.K. & K.C. Tsui (2002). A Multivariate Generalized Autoregressive Conditional Heteroscedasticity model with time-varying correlations, Journal of Business and Economic Statistics, 20, 351-362.
- Wang, M., & Yao, Q. (2005). Modeling multivariate volatilities: an ad hoc approach, to appear in "Contemporary Multivariate Analysis and Experimental Designs" J. Fan, G. Li& R. Li (edit.) World Scientific, Singapore.
- Yan, J. (2005). Asymmetry, fat-tail, and autoregressive conditional density in financial return data with systems of frequency curves. University of Iowa, working paper.