

Linear Programming: Solution

Problem 1

Consider the following linear program:

$$\text{Max } Z = 8X_1 + 10X_2$$

constraints

$$X_2 \leq 100$$

$$X_1 \geq 40$$

$$X_1 + 2X_2 \geq 160$$

$$X_1 - 3X_2 \leq 60$$

$$X_1 + X_2 \leq 200$$

- Solve the problem using the graphical solution
- What is the value of the objective function

Problem 2

$Z = 8, X_1 = 2, X_2 = 0$ is the optimal solution to the following LP:

$$\text{max } Z = 4X_1 + X_2$$

s.t.

$$3X_1 + X_2 \leq 6$$

$$5X_1 + 3X_2 \leq 15$$

$$X_1, X_2 \geq 0$$

Use the graphical approach to answer the following questions:

- Determine the range of values of C_1 for which the current basis remains optimal
- Determine the range of values of C_2 for which the current basis remains optimal
- Determine the range of values of b_1 for which the current basis remains optimal
- Determine the range of values of b_2 for which the current basis remains optimal

Problem 3

Use the Simplex algorithm to find the optimal solution to the following problems.

1. $\text{Max } Z = 2X_1 - X_2 + X_3$

constraints

$$3X_1 + X_2 + X_3 \leq 6$$

$$X_1 - X_2 + 2X_3 \leq 1$$

$$X_1 + X_2 - X_3 \leq 3$$

$$X_1, X_2, X_3 \geq 0$$

2. $\text{Min } Z = 4X_1 - X_2$

constraints

$$2X_1 + X_2 \leq 8$$

$$X_2 \leq 5$$

$$X_1 - X_2 \leq 4$$

$$X_1, X_2 \geq 0$$

Problem 4

The following table is the optimal tableau for a maximization problem and the constraints are (\leq)

X_1	X_2	X_3	X_4	X_5	S_1	S_2	S_3	
0	0	1	-4	1	1	0	0	10
-5	1	0	4	5	0	1	0	5
-3	0	6	1	9	0	0	1	5
-1	3	7	-5	8	0	0	0	0

Formulate the problem and write the optimal tableau.

Problem 5

Consider the LP program

$$\text{Max } Z = 2 X_1 + 3 X_2$$

Constraints

$$X_1 + X_2 \leq 10$$

$$2X_1 + X_2 \geq 4$$

$$X_1 + 3X_2 \leq 24$$

$$2X_1 + X_2 \leq 16$$

$$X_1, X_2 \geq 0$$

1. Solve this problem using the graphical solution procedure.
2. Compute the range of optimality for c_1
3. Compute the range of optimality for c_2
4. Suppose that c_1 is increased from 2 to 2.5. What is the new optimal solution?
5. Suppose that c_2 is increased from 3 to 1. What is the new optimal solution?