Sensitivity Analysis and Duality Using Solver

Problem 1
A television manufacturing company is producing four television models: M1, M2, M3, M4. Each TV set to be produced is undergoing two phases. Assembly and testing. The times required for assembly and for testing a unit of each of the 4 models, the total time available for assembly and for testing, as well as the unit profits per model are given in the following table:

	M1	**M2**	**M3**	**M4**	**Total available time monthly**
Assembly (in hrs)	8	12	10	13	2200
Testing (in hrs)	1	3	4	5	700
Profit	€40	€80	€90	€100	

In addition the manufacturing company has an overall capacity limitation of 180 units, monthly, i.e. no more than 180 units totally can be produced per month. Finally, the total assembly for models M3 and M4, cannot exceed 100 units monthly.
The production manager is interested in developing the optimal production plan, that will maximize the company's total profit.
He formulates the problem as an LP, as follow:

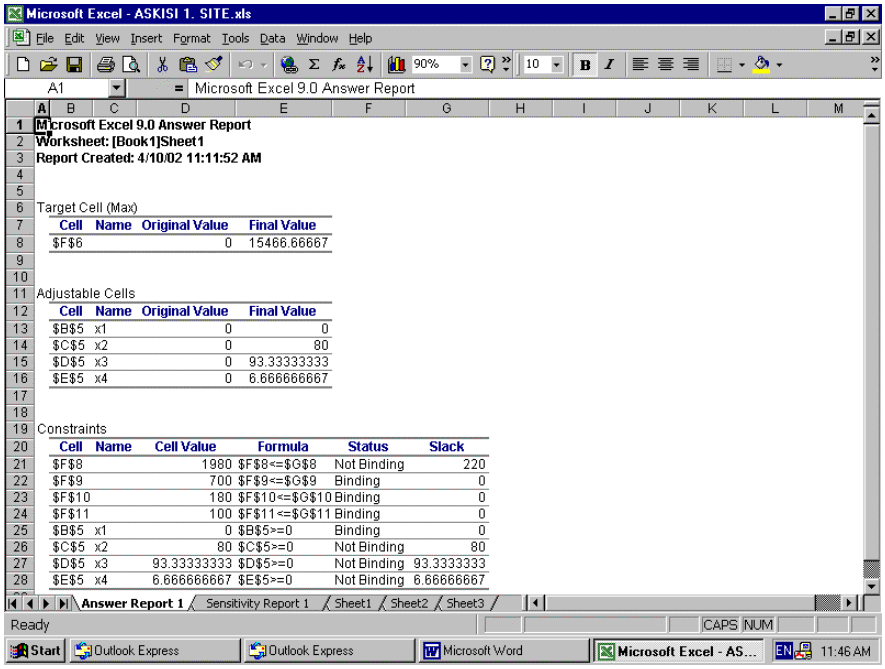
Let xi = number of units produced for model Mi (I=1, 2, 3, 4).

MaxZ = 40x1+80x2+90x3+100x4

Subject to the constraints:

$8x1+12x2+10x3+13x4 \leq 2200$ (1)
$x1+3x2+4x3+5x4 \leq 700$ (2)
$x1+x2+x3+x4 \leq 180$ (3)
$x3+x4 \leq 100$ (4)
$x1, x2, x3, x4 \geq 0$

Please answer the following questions:



Microsoft Excel 9.0 Answer Report
Worksheet: [Book1]Sheet1
Report Created: 4/10/02 11:11:52 AM

Target Cell (Max)

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$F$6		0	15466.66667

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	x1	0	0
$C$5	x2	0	80
$D$5	x3	0	93.33333333
$E$5	x4	0	6.666666667

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
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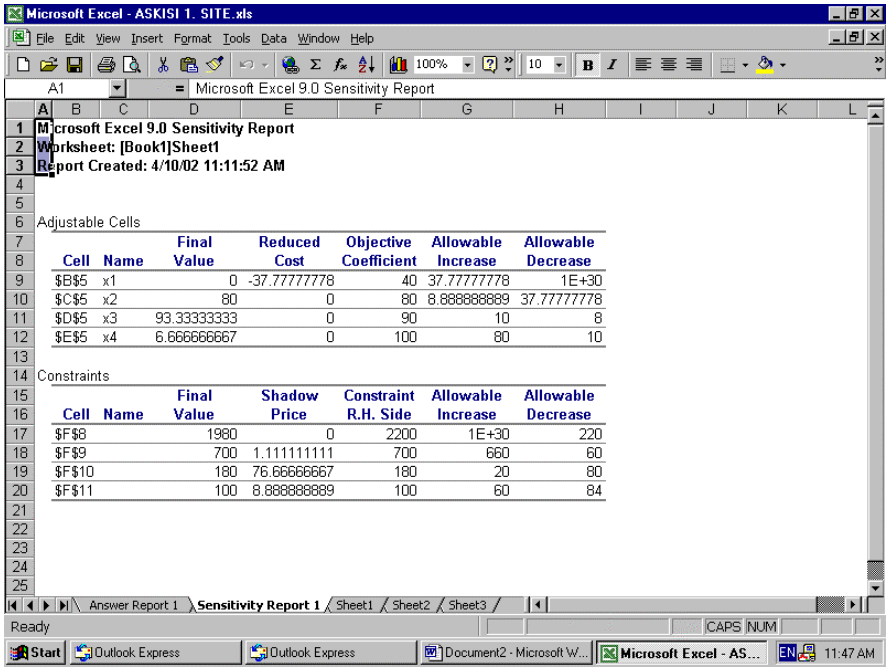
$F$9		700	$F$9<=$G$9	Binding	0
$F$10		180	$F$10<=$G$10	Binding	0
$F$11		100	$F$11<=$G$11	Binding	0
$B$5	x1	0	$B$5>=0	Binding	0
$C$5	x2	80	$C$5>=0	Not Binding	80
$D$5	x3	93.33333333	$D$5>=0	Not Binding	93.3333333
$E$5	x4	6.666666667	$E$5>=0	Not Binding	6.66666667



Microsoft Excel 9.0 Sensitivity Report
Worksheet: [Book1]Sheet1
Report Created: 4/10/02 11:11:52 AM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	x1	0	-37.77777778	40	37.77777778	1E+30
$C$5	x2	80	0	80	8.888888889	37.77777778
$D$5	x3	93.33333333	0	90	10	8
$E$5	x4	6.666666667	0	100	80	10

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$8		1980	0	2200	1E+30	220
$F$9		700	1.111111111	700	660	60
$F$10		180	76.66666667	180	20	80
$F$11		100	8.888888889	100	60	84

1. What is the optimal production plan? What is the company's total expected profit? Explain the dual price of constraint (3).
2. Which model is not to be produced? Why? What should happen in order to start producing it?
3. What will happen to the production plan and to the total profit if the unit profit from model M3 is increased by €20?
4. What will happen to the production plan and to the total profit if the time available for assembly is reduced by 50 hours?
5. The marketing department is recommending the production of a new model which would require 10 hours of assembly, 10 hours of testing, and would contribute a unit profit of €35. Is it worth it to produce the new model?

Problem 2
Steelco uses coal, iron, and labor to produce three types of steel. The inputs (and sales price) for one ton of each type of steel are shown in the table below.

	Coal	Iron	Labor	Sales price
Steel 1	3 tons	1 tons	1 hour	51€
Steel 2	2 tons	0 tons	1 hour	30€
Steel 3	1 tons	1 tons	1 hour	25€

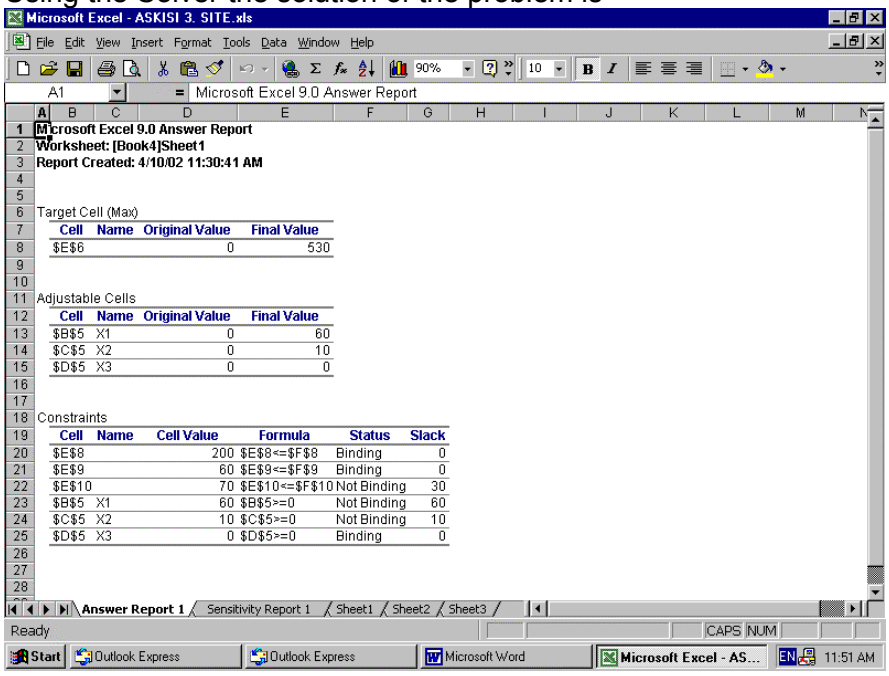
Up to 200 tons of coal can be purchased at a price of 10€ per ton.
Up to 60 tons of iron can be purchased at a price of 8€ per ton.
Up to 100 labor hours can be purchased at 5€ per hour.
The company wants to maximize the total profit.

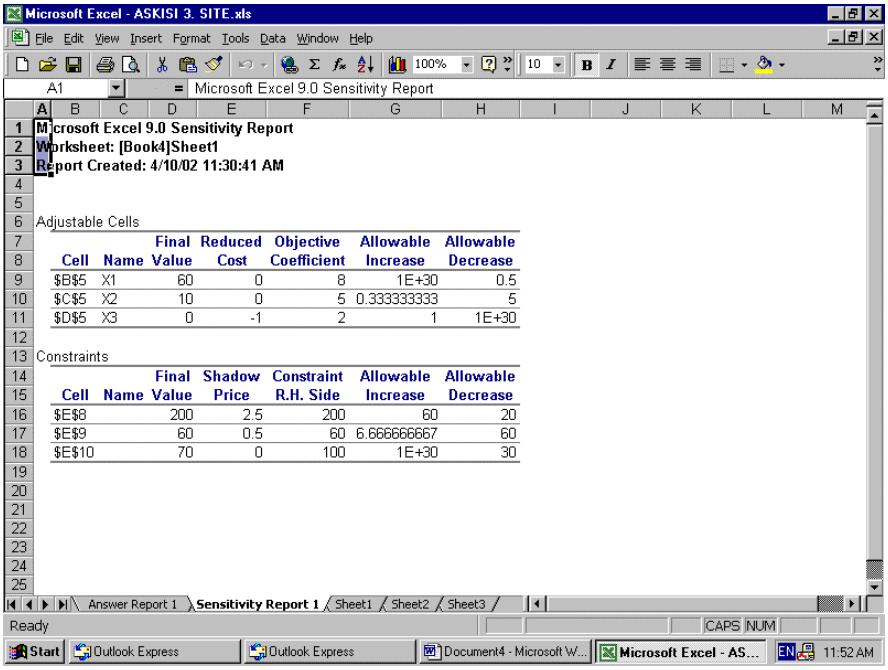
Let:

X_1 = Tons of steel 1 produced
X_2 = Tons of steel 2 produced
X_3 = Tons of steel 3 produced

The objective function is:

MaxZ =8X1+5X2+2X3
Subject to the constraints:
$3x1+2x2+x3 \leq 200$ (1)
$x1+x3 \leq 60$ (2)
$x1+x2+x3 \leq 100$ (3)
$x1, x2, x3 \geq 0$
Using the Solver the solution of the problem is





Microsoft Excel 9.0 Sensitivity Report
Worksheet: [Book4]Sheet1
Report Created: 4/10/02 11:30:41 AM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	X1	60	0	8	1E+30	0.5
$C$5	X2	10	0	5	0.333333333	5
$D$5	X3	0	-1	2	1	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$8		200	2.5	200	60	20
$E$9		60	0.5	60	6.666666667	60
$E$10		70	0	100	1E+30	30

Please answer the following questions:

1. What would profit be if only 40 tons of iron could be purchased?
2. What is the smallest price per ton for steel 3 that would make it desirable to produce it?
3. Find the new optimal solution if steel 1 sold for 55€ per ton.

Problem 3

Giapetto, Inc., sells wooden soldiers and wooden trains. The resources used to produce a soldier and train are shown in the table below.

	Soldier	**Train**
Lumber	3 board feet	5 board feet
Labor	2 hours	4 hours

A total of 145,000 board feet of lumber and 90,000 hours of labor are available. Up to 50,000 soldiers and up to 50,000 trains can be sold, with trains selling for 55€ and soldiers for 32€. In addition to producing trains and soldiers itself, Giapetto can buy (from outside supplier) extra soldiers at 27€ each and extra trains at 50€ each.

Let:

SM = thousands of soldiers manufactured
SB = thousands of soldiers bought at 27€
TM = thousands of trains manufactured
TB = thousands of trains bought at 50€

The objective function is:

Max Z= 32SM+55TM+5SB+5TB

Subject to the constraints:

$3SM+5TM \leq 145$ (1)

$2SM+4TM \leq 90$ (2)

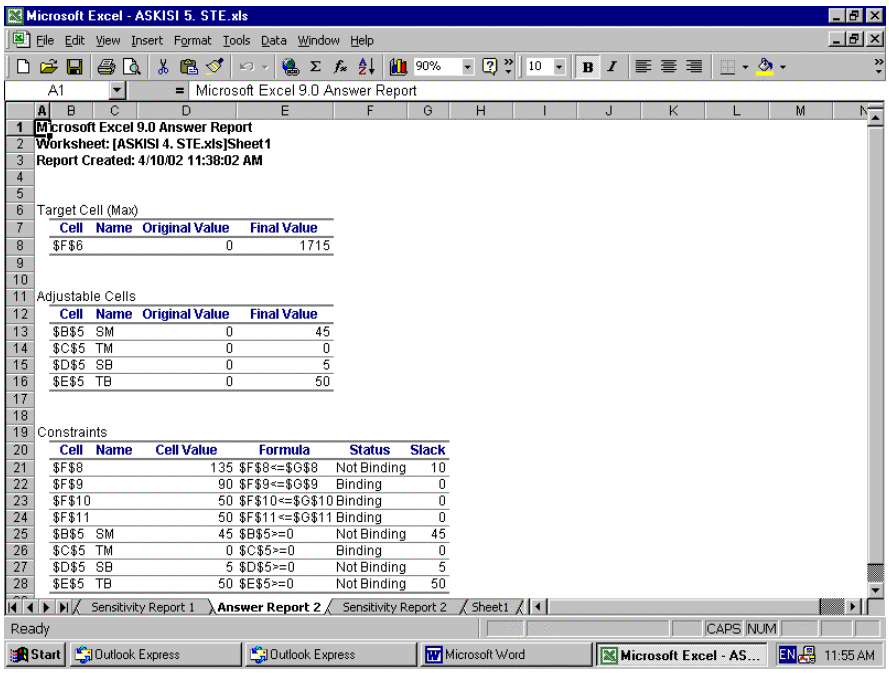
$SM+SB \leq 50$ (3)

$TM+TB \leq 50$ (4)

$SM, TM, SB, TB \geq 0$

Using the Excel the solution of the problem is

Please answer the following questions:



Microsoft Excel - ASKISI 5. STE.xls

Microsoft Excel 9.0 Answer Report
Worksheet: [ASKISI 4. STE.xls]Sheet1
Report Created: 4/10/02 11:38:02 AM

Target Cell (Max)

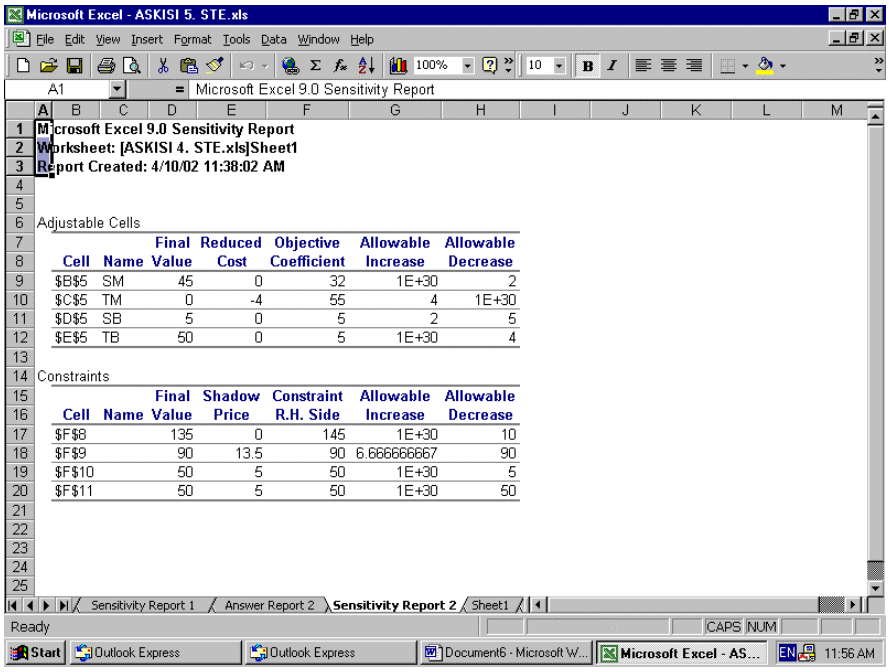
Cell	Name	Original Value	Final Value
$F$6		0	1715

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	SM	0	45
$C$5	TM	0	0
$D$5	SB	0	5
$E$5	TB	0	50

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$F$8		135	$F$8<=$G$8	Not Binding	10
$F$9		90	$F$9<=$G$9	Binding	0
$F$10		50	$F$10<=$G$10	Binding	0
$F$11		50	$F$11<=$G$11	Binding	0
$B$5	SM	45	$B$5>=0	Not Binding	45
$C$5	TM	0	$C$5>=0	Binding	0
$D$5	SB	5	$D$5>=0	Not Binding	5
$E$5	TB	50	$E$5>=0	Not Binding	50



Microsoft Excel - ASKISI 5. STE.xls

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [ASKISI 4. STE.xls]Sheet1
Report Created: 4/10/02 11:38:02 AM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	SM	45	0	32	1E+30	2
$C$5	TM	0	-4	55	4	1E+30
$D$5	SB	5	0	5	2	5
$E$5	TB	50	0	5	1E+30	4

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$F$8		135	0	145	1E+30	10
$F$9		90	13.5	90	6.666666667	90
$F$10		50	5	50	1E+30	5
$F$11		50	5	50	1E+30	50

1. If Giapetto could purchase trains for 48€ per train, what would be the new optimal solution to the LP? Explain
2. What is the most Giapetto would be willing to pay for another 100 board feet of lumber? For another 100 hours of labor?
3. If 60,000 labor hours are available, what would Giapetto's profit be?
4. If only 40,000 trains could be sold, what would Giapetto's profit be?

Problem 4

A regional development company owning 1500 acres of prime land, has undertaken to build three types (A, B, C) of family homes.

The number of single family A homes must be at least 50% of the total.

Minimum lot sizes of 1, 2 and 3 acres are required for the three types of family homes, respectively. In addition, it is estimated that 15% of the acreage will be consumed for the construction of the streets and the easement for utilities. The company must construct the network of water services for the area. The maximum cost of construction of the water service network is €700,000. The supply of water through the network is limited to 270.000 litters (lt) per day.

The following data summarize the cost of construction of the water service network, as well as the water consumption (for an average size family).

Types family homes	Cost of connection to water service network per unit (€)	Water consumption daily per unit (lt)
A	1200	400
B	1500	600
C	2000	850

The estimated returns from the different housing units are:

Types	Net profit per unit (€)
A	15,000
B	20,000
C	30,000

Assuming that any number of houses built of any type can be sold, the company must decide the number of units to be constructed of each housing type to maximize the total profit.

The problem can be formulated as the following linear program.

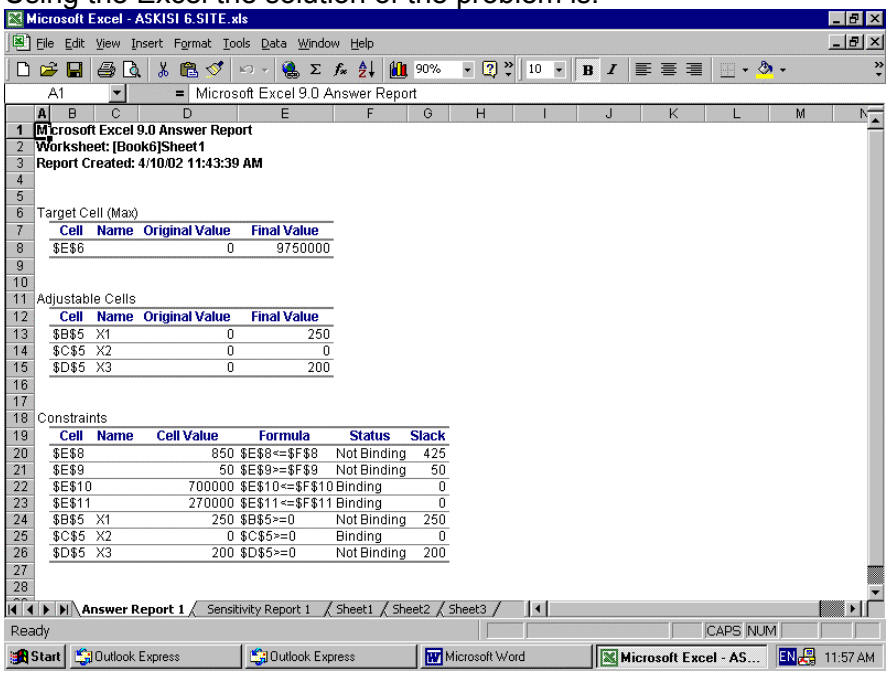
Let:

X_1=number of units of type A
X_2=number of units of type B
X_3=number of units of type C

Max $Z=15{,}000X_1+20{,}000X_2+30{,}000X_3$
Subject to the constraints:

$X_1+2X_2+3X_3 \leq 1275$ (1)
$X_1-X_2-X_3 \geq 0$ (2)
$1200X_1+1500X_2+2000X_3 \leq 700{,}000$ (3)
$400X_1+600X_2+850X_3 \leq 270.000$ (4)
$X_1, X_2, X_3 \geq 0$

Using the Excel the solution of the problem is:



Microsoft Excel - ASKISI 6.SITE.xls

Microsoft Excel 9.0 Answer Report
Worksheet: [Book6]Sheet1
Report Created: 4/10/02 11:43:39 AM

Target Cell (Max)

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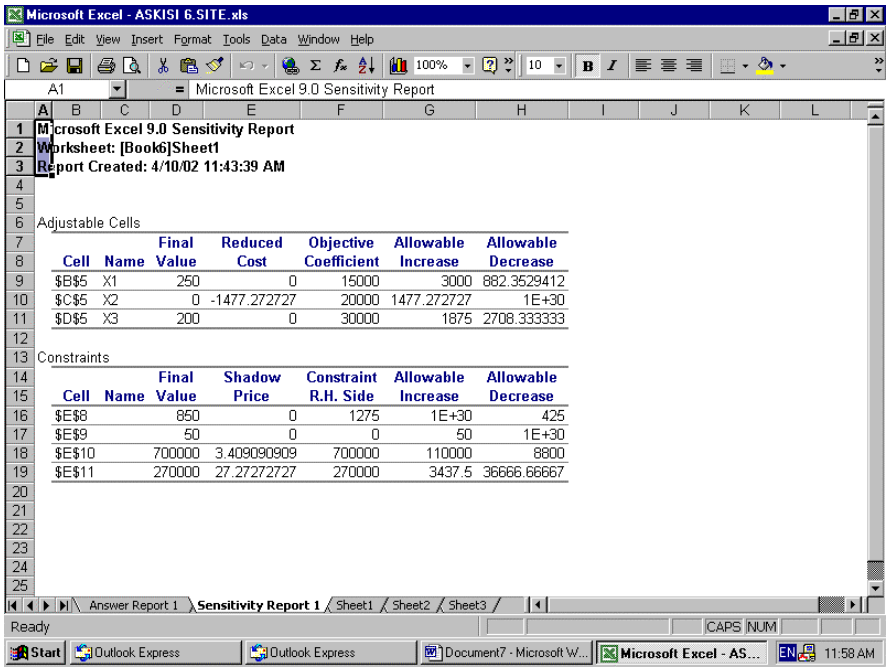
Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	X1	0	250
$C$5	X2	0	0
$D$5	X3	0	200

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
$E$8		850	$E$8<=$F$8	Not Binding	425
$E$9		50	$E$9>=$F$9	Not Binding	50
$E$10		700000	$E$10<=$F$10	Binding	0
$E$11		270000	$E$11<=$F$11	Binding	0
$B$5	X1	250	$B$5>=0	Not Binding	250
$C$5	X2	0	$C$5>=0	Binding	0
$D$5	X3	200	$D$5>=0	Not Binding	200



Microsoft Excel - ASKISI 6.SITE.xls

Microsoft Excel 9.0 Sensitivity Report
Worksheet: [Book6]Sheet1
Report Created: 4/10/02 11:43:39 AM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	X1	250	0	15000	3000	882.3529412
$C$5	X2	0	-1477.272727	20000	1477.272727	1E+30
$D$5	X3	200	0	30000	1875	2708.333333

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$8		850	0	1275	1E+30	425
$E$9		50	0	0	50	1E+30
$E$10		700000	3.409090909	700000	110000	8800
$E$11		270000	27.27272727	270000	3437.5	36666.66667

Please answer to the following questions:

a) How many units of housing types (A, B and C) should be build? What is the total profit? Explain the constraints (1) and (2). Explain the dual price for constraint (3).
b) Which types of family homes cannot be built? Why? What would you recommend that should be done in order the company to decide to build them.
c) What happens to the construction plan if the profit of each unit of type A increases by €2,000? What happens to the total profit?
d) If the budget available for water service increases to €750,000 what happens to the construction plan, and to the total profit?
e) If the maximum water consumption decreases by 20.000 lt what happens to the construction plan, and to the total profit?

f) The company has developed plans for a new type of house (D) which requires 4 acres, the water service connection costs €3,000 and the water consumption is 1000lt. The net return for this house is €45,000. Must this type of house be constructed? Please explain your answer.

Problem 5

A company produces three types of tools (A, B, C).
Each tool A requires 9 kilos of iron, 50 minutes machine time, and 60 minutes labor.
Each tool B requires 8 kilos of iron, 110 minutes machine time, and 40 minutes labor.
Finally, each tool C requires 7.5 kilos of iron, 90 minutes machine time, and 55 minutes labor.
The company works on a weekly schedule of 5 days, with 2 shifts of 7.5 hours each.
It has 4 machines available for production and 15 employees on each shift.
The marketing department requires that at least 300 tools of all types be produced each week. The profit of each tool A, B, and C is €12, €15, and €13.5, respectively.
The company's capacity is 3 tons of iron.
The company would like a production plan that will maximize profit.

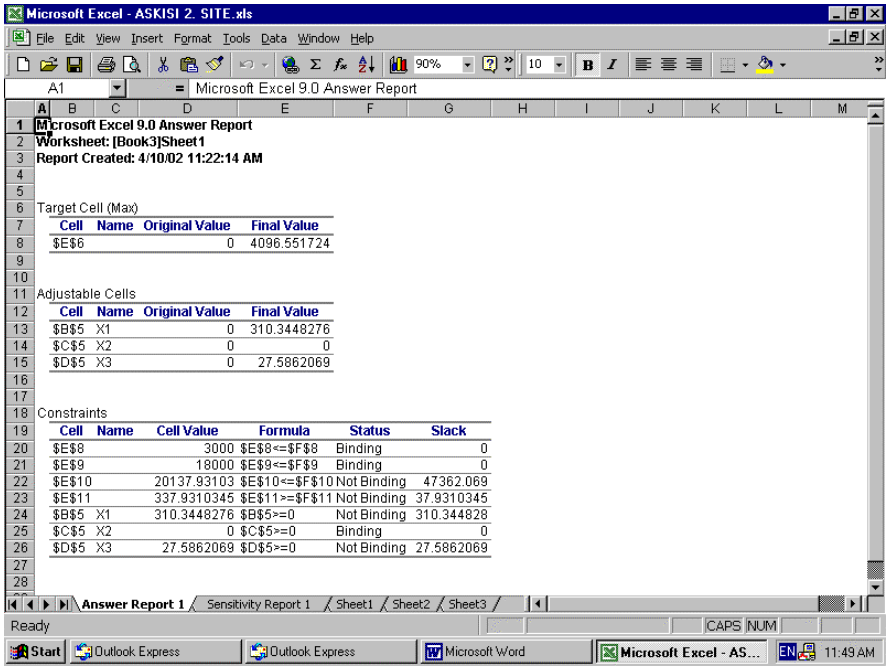
The problem can be formulated as the following linear program.

Let:
X_1=number of tools A
X_2=number of tools B
X_3=number of tools C

The objective function is:

$MaxZ=12X_1+15X_2+13.5X_3$
Subject to the constraints:
$9X_1+8X_2+7.5X_3 \leq 3{,}000$ (1)
$50X_1+110X_2+90X_3 \leq 18{,}000$ (2)
$60X_1+40X_2+55X_3 \leq 67{,}500$ (3)
$X_1+X_2+X_3 \geq 300$ (4)
$X_1, X_2, X_3 \geq 0$

Using the Excel the solution of the problem is



Microsoft Excel 9.0 Answer Report
Worksheet: [Book3]Sheet1
Report Created: 4/10/02 11:22:14 AM

Target Cell (Max)

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$E$6		0	4096.551724

Adjustable Cells

Cell	Name	Original Value	Final Value
$B$5	X1	0	310.3448276
$C$5	X2	0	0
$D$5	X3	0	27.5862069

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
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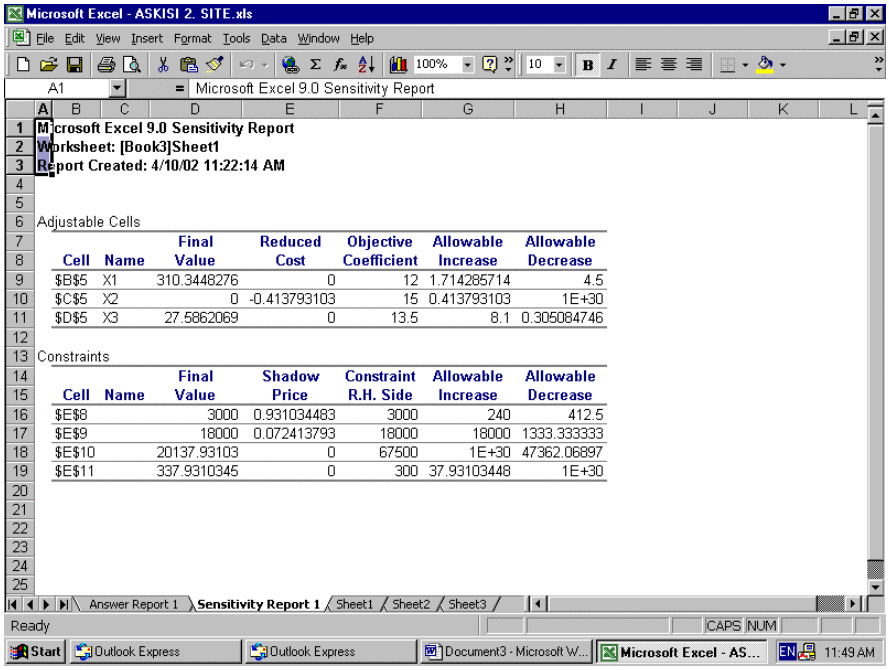
$E$9		18000	$E$9<=$F$9	Binding	0
$E$10		20137.93103	$E$10<=$F$10	Not Binding	47362.069
$E$11		337.9310345	$E$11>=$F$11	Not Binding	37.9310345
$B$5	X1	310.3448276	$B$5>=0	Not Binding	310.344828
$C$5	X2	0	$C$5>=0	Binding	0
$D$5	X3	27.5862069	$D$5>=0	Not Binding	27.5862069



Microsoft Excel 9.0 Sensitivity Report
Worksheet: [Book3]Sheet1
Report Created: 4/10/02 11:22:14 AM

Adjustable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
$B$5	X1	310.3448276	0	12	1.714285714	4.5
$C$5	X2	0	-0.413793103	15	0.413793103	1E+30
$D$5	X3	27.5862069	0	13.5	8.1	0.305084746

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
$E$8		3000	0.931034483	3000	240	412.5
$E$9		18000	0.072413793	18000	18000	1333.333333
$E$10		20137.93103	0	67500	1E+30	47362.06897
$E$11		337.9310345	0	300	37.93103448	1E+30

Please answer the following questions:

1. How many tools of types (A, B and C) should be produced? What is the total profit? Explain the constraints (2) and (3). Explain the dual price for constrain (2).
2. Which types of tools cannot be produced? What you recommend that should be done in order the company to decide to produced them?
3. What happens to the production plan if the profit of each tool C increases by to €2? What happens to the total profit?
4. If the company has available for production 20 employees on each shift, what happens to the production plan, and to the total profit?
5. If the available tons of iron increases by 200 kilos, what happens to the production plan, and to the total profit? What happens if the tons of iron decreases by 400 kilos?

6. The company has developed plans for a new type of tool D which requires 10 kilos of iron, 45 minutes machine time and 50 minutes labor. The net return for this type is €11. Must this type of tool be produced?