Linear Programming: Sensitivity Analysis and Duality (1)

Problem 1
The final optimal tableau of a maximization linear programming problem with constraints of type (≤) is
Where X_1, X_2 are the decision variables and S_1, S_2 and S_3 are the slack variables.

X_1	X_2	S_1	S_2	S_3	
0	1	½	-1/2	0	2
1	0	-1/8	3/8	0	3/2
0	0	1	-2	1	4
0	0	¼	¼	0	5

Suppose that it is decided to increase the right-hand side of one of the constrains. Which one do you recommend for expansion and why? What is the maximum amount of increase in this case? Find the corresponding new value of the objective function.

Problem 2
Storex manufactures three types of computer storage disk packs: M1, M2 and M3. Each disk pack must go through four different production phases: cutting, cleaning and coating, testing and assembly. We have the following set of information with which to make a product mix decision for the company:

	Processing hours M_1	M_2	M_3	**Total hours available per week**
Cutting	0.2	1.2	2.4	3200
Cleaning and coating	0.8	2.3	5	8000
Testing	0.2	0.8	1.2	2400
Assembly	0.3	1	2	4800
Profit margin	€ 50	150	320	

Let x1, x2 and x3, respectively, be the quantities of M_1, M_2, and M_3 to be produced, and let s_1, s_2, s_3 και s_4, respectively, be the slack variables representing the unused hours of the cutting, cleaning and coating, testing and assembly facilities. The simplex tableau for an optimum solution of the problem is given below:

X_1	X_2	X_3	S_1	S_2	S_3	S_4	
0	0	1	0,77	0,34	-2.15	0	68,97
1	0	0	-5,34	2,06	2,06	0	4413,79
0	1	0	0,17	-1,03	3,96	0	1793,10
0	0	0	-0,12	-0,27	-0,27	1	1544,83
0	0	0	6,89	58,62	8,62	0	511,72

1. What are the shadow prices of the four production processes? What do these shadow prices mean?
2. A competitor has just introduced a new disk pack. Each unit of this disk pack will require 3 hours of cutting, 5 hours of cleaning and coating, 3 hours of testing, and 4 hours of assembly.the profit margin on this new disk pack will be €350. Should Storex bring out the new type of disk pack?

Problem 3
Healthy Vitamin uses two substances, M1 and M2, to produce vitamin pills. Each pill must contain 30 units of vitamin A and 20 units of vitamin B. From one unit of M1 can be extracted 20 units of vitamin A and 10 units of vitamin B. From one unit of M2 can be extracted 10 units of vitamin A and 10 units of vitamin B. The unit costs are 3 for M1 and 2 for M2. The company wants to mix the two substances in such proportions as to minimize the total cost of a vitamin pill.
Let X and X_2, respectively, be the quantities of M1and M2 used a pill. Then the problem formulated as follows:

min $Z=30X_1+20X_2$
constraints
$20X_1+10X_2\geq30(1)$
$10X_1+10X_2\geq20(2)$
$X_1, X_2 \geq0$

The optimal simplex tableau is the following:

X_1	X_2	S_1	S_2	
1	0	-0,1	0,1	1
0	1	0,1	-0,2	1
0	0	0,1	0,1	-5

a. what are the shadow prices of vitamins A and B?
b. suppose the vitamin A requirement in a pill is increased by one unit. How will the unit cost of the pill be affected?
c. suppose the company can purchase vitamin B in pure form at a cost of 3c per 10 units. Should it use any vitamin B in pure form?
d. The company can use another substance, say Y. from one unit of Y can be extracted 10 units of A and 20 units of B. the cost of Y is 4 c per unit. Should the company use any of this substance in its vitamin production? Why?

Problem 4
Sugarco can manufacture three types of candy bar. Each candy bar consists totally of sugar and chocolate. The composition of each type of candy bar and the profit earned from each candy bar are shown in the table below

	Amount of sugar (ounces)	**Amount of chocolate (ounces)**	**Profit (€)**
Candy bar 1	1	2	3
Candy bar 2	1	3	7
Candy bar 3	1	1	5

50 oz of sugar and 100 oz of chocolate are available. After defining x_i to be the number of type i candy bars manufactured, Sugarco should solve the following LP.

Max $Z = 3x_1 + 7x_2 + 5x_3$

$x_1 + x_2 + x_3 \leq 50$ (Sugar constraint)
$2x_1 + 3x_2 + x_3 \leq 100$ (Chocolate constraint)
$x_1, x_2, x_3 \geq 0$

After adding slack variables s_1 and s_2, the optimal tableau is shown in the table below

Z	**x_1**	**X_2**	**x_3**	**s_1**	**s_2**	**rhs**	**Basic variable**
1	3	0	0	4	1	300	Z = 300
0	$\frac{1}{2}$	0	1	$\frac{3}{2}$	$-\frac{1}{2}$	25	$x_1 = 25$
0	$\frac{1}{2}$	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	25	$x_2 = 25$

Using this optimal tableau answer the following questions

a. For what values of type 1 candy bar profit does the current basis remain optimal? If the profit for a type 1 candy bar were 7€, what would be the new optimal solution to Sugarco's problem?
b. For what values of type 2 candy bar profit would the current basis remain optimal? If the profit for a type 2 candy bar were 13€ what would be the new optimal solution to Sugarco's problem?
c. For what amount of available sugar would the current basis remain optimal?
d. If 60 oz of sugar were available, what would be Sugarco's profit? How many of each candy bar should they make? Could these questions be answered if only 30 oz of sugar were available?
e. Suppose a type 1 candy bar used only 0.5 oz of sugar and 0.5 oz of chocolate. Should Sugarco now make type 1 candy bars?
f. Sugarco is considering making type 4 candy bars. A type 4 candy bar earns 17€ profit and requires 3 oz of sugar and 4 oz of chocolate. Should Sugarco manufacture any type 4 candy bars?

Problem 5

Gapco has a daily budget of 320 hours of labor and 350 units of raw material to manufacture 2 products. If necessary, the company can employ up to 10 hours daily of overtime labor hours at the additional cost of 2€ per hour. It takes 1 labor hour and 3 units of raw material to produce 1 unit of product 1 and 2 labor hours and 1 unit of raw material to produce 1 unit of product 2. The profit per unit of product 1 is 10€ and that of product 2 is 12€. Let X_1 and X_2 define the daily number of units produced of products 1 and 2 and X_3 the daily hours of overtime used. The LP model and its associated optimal Simplex tableau are then given as below.

Maximize $Z = 10 X_1 + 12 X_2 - 2 X_3$

Subject to:

$X_1 + 2 X_2 - X_3 \leq 320$ (labor hours)
$3X_1 + X_2 \leq 350$ (Raw material)

$X_3 \leq 10$ (Overtime)
$X_1, X_2, X_3 \geq 0$

Basic	**X_1**	**X_2**	**X_3**	**S_1**	**S_2**	**S_3**	**Solution**
Z	0	0	0	26/5	8/5	16/5	2256
X_2	0	1	0	3/5	-1/5	3/5	128
X_1	1	0	0	-1/5	2/5	-1/5	74
X_3	0	0	1	0	0	1	10

1. Determine the optimal solution of the problem
2. Determine the dual prices and the applicability ranges of their associated resources
3. Examine the dual prices for labor hours (constraint 1) and overtime hours (constraint 3). Shouldn't these 2 values be the same? Explain.
4. Gapco currently pays an additional 2€ per overtime hour. What is the most the company should be willing to pay?
5. If Gapco can acquire additional 100 units of raw material daily at 1.50€ a unit, would you advice the company to do so? What if the cost of raw material is 2 € a unit?
6. Suppose that Gapco is experiencing shortage in raw material and that it cannot acquire more than 200 units a day, determine the associated optimal solution.
7. Suppose that Gapco can use no more than 8 hours of overtime daily, find the new optimum solution.